$\boxed{7.1}$ INTEGRATION BY PARTS

Recall: PRODUCT RULE

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\,g'(x) + g(x)\,f'(x)$$

If we integrate:

$$f(x)\,g(x) = \int \left[f(x)g'(x) + g(x)f'(x)\right] dx$$

$$= \int f(x)\,g'(x)\,dx + \int g(x)\,f'(x)\,dx$$

$$\boxed{\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int g(x)\,f'(x)\,dx}$$

FORMULA OF INTEGRATION BY PARTS

$u = f(x) \qquad du = f'(x)\,dx$

$v = g(x) \qquad dv = g'(x)\,dx$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int \underbrace{x}_{u} \underbrace{\sin x \, dx}_{dv}$$

we choose:

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$$= x(-\cos x) - \int (-\cos x) \, dx$$

$$= -x\cos x + \int \cos x \, dx$$

$$= \underline{-x\cos x + \sin x + C}$$

Our choice of u and dv gave the integral $\int \cos x \, dx$

which is EASIER than $\int x \sin x \, dx$

<u>WARNING</u>:

$$\int x \sin x \, dx = \int \underbrace{(\sin x)}_{u} \cdot \underbrace{x \, dx}_{dv} = (*)$$

$u = \sin x \qquad dv = x \, dx$

$du = \cos x \qquad v = \frac{1}{2} x^2$

$$(*) = \frac{1}{2} x^2 \sin x - \int \frac{1}{2} x^2 \cos x \, dx$$

THIS IS HARDER!

Our choice was BAD.

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Example:-

$$\int \ln x\,dx$$

PARTS:

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$$= x\ln x - \int x\,\frac{1}{x}\,dx$$

$$= x\ln x - \int dx$$

$$= x\ln x - x + C$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

<u>Example:-</u>

$$\int t^2 e^t\,dt \qquad \left| \begin{array}{ll} u = t^2 & dv = e^t\,dt \\ du = 2t\,dt & v = e^t \end{array} \right.$$

$$= t^2 e^t - 2\int t e^t\,dt \qquad \left| \begin{array}{ll} u = t & dv = e^t\,dt \\ du = dt & v = e^t \end{array} \right.$$

$$= t^2 e^t - 2\left[t e^t - \int e^t\,dt \right]$$

$$= t^2 e^t - 2te^t + 2\int e^t\,dt$$

$$= \underline{t^2 e^t - 2te^t + 2e^t + C}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Example:-

$$I = \int e^x \sin x \, dx$$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x \, dx \qquad v = -\cos x$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x \, dx \qquad v = \sin x$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx = I$$

$$I = -e^x \cos x + e^x \sin x - I$$

$$2I = -e^x \cos x + e^x \sin x$$

$$I = \frac{1}{2}\left(-e^x \cos x + e^x \sin x\right)$$

$$\underline{\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) + C}$$