### Strategy for Evaluating $\int \sin^m x \cos^n x \, dx$

(a) If the power of cosine is odd ($n = 2k + 1$), save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine:

$$\int \sin^m x \cos^{2k+1} x \, dx = \int \sin^m x \, (\cos^2 x)^k \cos x \, dx$$

$$= \int \sin^m x \, (1 - \sin^2 x)^k \cos x \, dx$$

Then substitute $u = \sin x$.

(b) If the power of sine is odd ($m = 2k + 1$), save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine:

$$\int \sin^{2k+1} x \cos^n x \, dx = \int (\sin^2 x)^k \cos^n x \sin x \, dx$$

$$= \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx$$

Then substitute $u = \cos x$. [Note that if the powers of both sine and cosine are odd, either (a) or (b) can be used.]

(c) If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

It is sometimes helpful to use the identity

$$\sin x \cos x = \tfrac{1}{2} \sin 2x$$

$$\int \cos^3 x \, dx =$$

$$= \int (\cos^2 x)(\cos x)\, dx$$

$$= \int (1 - \sin^2 x)(\cos x)\, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int (1 - u^2)\, du$$

$$= u - \frac{u^3}{3} + C$$

$$= \underline{\sin x - \frac{1}{3}\sin^3 x + C}$$

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$$\int \sin^5 x \cos^2 x \, dx$$

$$= \int \sin^4 x \cos^2 x \sin x \, dx$$

$$= \int (\sin^2 x)^2 \cos^2 x \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \cos^2 x \sin x \, dx \quad \left| \begin{array}{l} u = \cos x \\ du = -\sin x dx \end{array} \right.$$

$$= -\int (1-u^2)^2 u^2 \, du$$

$$= -\int (1-2u^2+u^4) u^2 \, du$$

$$= -\int (u^2-2u^4+u^6) \, du$$

$$= -\left( \frac{u^3}{3} - 2\frac{u^5}{5} + \frac{u^7}{7} \right) + C$$

$$= -\left( \frac{1}{3}\cos^3 x - \frac{2}{5}\cos^5 x + \frac{1}{7}\cos^7 x \right) + C$$