TRIG INTEGRALS

Recall:- $\int \sin^a x \cos^b x \, dx$

a or b are ODD

USED $\cos^2 x + \sin^2 x = 1$

(a) $\int F(\sin x) \cos x \, dx$ $u = \sin x$

(b) $\int G(\cos x) \sin x \, dx$ $u = \cos x$

Recall:- $\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

Strategy for Evaluating $\int \tan^m x \sec^n x \, dx$

(a) If the power of secant is even ($n = 2k$, $k \geqslant 2$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\int \tan^m x \sec^{2k} x \, dx = \int \tan^m x \, (\sec^2 x)^{k-1} \sec^2 x \, dx$$

$$= \int \tan^m x \, (1 + \tan^2 x)^{k-1} \sec^2 x \, dx$$

Then substitute $u = \tan x$.

(b) If the power of tangent is odd ($m = 2k + 1$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\int \tan^{2k+1} x \sec^n x \, dx = \int (\tan^2 x)^k \sec^{n-1} x \, \sec x \, \tan x \, dx$$

$$= \int (\sec^2 x - 1)^k \sec^{n-1} x \, \sec x \, \tan x \, dx$$

Then substitute $u = \sec x$.

$$\int \tan^5 x \sec^7 x \, dx$$

$$= \int \tan^4 x \sec^6 x \sec x \tan x \, dx$$

$$= \int \underbrace{(\sec^2 x - 1)^2 \sec^6 x}_{G(\sec x)} \underbrace{\sec x \tan x \, dx}_{d(\sec x)}$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int (u^2 - 1)^2 u^6 \, du$$

$$= \dots\dots\dots =$$

$$= \frac{1}{11} u^{11} - \frac{2}{9} u^9 + \frac{1}{7} u^7 + C$$

$$= \frac{1}{11} \sec^{11} x - \frac{2}{9} \sec^9 x + \frac{1}{7} \sec^7 x + C$$