Strategy for Evaluating $\int \tan^m x \sec^n x\, dx$

(a) If the power of secant is even ($n = 2k$, $k \geqslant 2$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\int \tan^m x \sec^{2k} x\, dx = \int \tan^m x\, (\sec^2 x)^{k-1} \sec^2 x\, dx$$

$$= \int \tan^m x\, (1 + \tan^2 x)^{k-1} \sec^2 x\, dx$$

Then substitute $u = \tan x$.

(b) If the power of tangent is odd ($m = 2k + 1$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\int \tan^{2k+1} x \sec^n x\, dx = \int (\tan^2 x)^k \sec^{n-1} x\, \sec x\, \tan x\, dx$$

$$= \int (\sec^2 x - 1)^k \sec^{n-1} x\, \sec x\, \tan x\, dx$$

Then substitute $u = \sec x$.

$$\int \tan^3 x \, dx$$

For other trig integrals:

NO CLEAR STRATEGY ☹

<u>TRY</u> → PARTS
→ TRIG IDENTITIES

<u>HELPFUL</u>

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \tan^3 x \, dx \qquad \tan^2 x = \sec^2 x - 1$$

$$= \int \tan x \tan^2 x \, dx$$

$$= \int \tan x \left(\sec^2 x - 1 \right) dx$$

$$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$$

$$u = \tan x$$
$$du = \sec^2 x \, dx$$

$$= \int u \, du - \int \tan x \, dx$$

$$= \frac{u^2}{2} - \ln |\sec x| + C$$

$$= \frac{1}{2} \tan^2 x - \ln |\sec x| + C$$

$$\boxed{\int \sec^3 x \, dx}$$

$$= \int (\sec x)(\sec^2 x)\, dx$$

$u = \sec x \qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \, dx$

$v = \tan x$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$\tan^2 x = \sec^2 x - 1$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1)\, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$= \sec x \tan x - \boxed{\int \sec^3 x \, dx} + \ln|\sec x + \tan x|$$

$$\int \sec^3 x \, dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C$$