Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$ or $\pi \leqslant \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

$$I = \int_0^{3\sqrt{2}/4} \frac{y^2}{(9-4y^2)^{3/2}}\, dy = \int_0^{3\sqrt{2}/4} \frac{y^2}{\left(\sqrt{3^2-(2y)^2}\right)^3}\, dy$$

$$2y = 3\sin\theta \qquad y = \frac{3}{2}\sin\theta$$

$$dy = \frac{3}{2}\cos\theta\, d\theta \qquad y = 0 \implies \theta = 0$$

$$y = \frac{3\sqrt{2}}{4} = \frac{3}{2}\,\frac{\sqrt{2}}{2} \implies \theta = \frac{\pi}{4}$$

$$\left(\sqrt{3^2-(2y)^2}\right)^3 = \left(\sqrt{3^2-3^2\sin^2\theta}\right)^3$$

$$= \left(3\sqrt{1-\sin^2\theta}\right)^3 = (3\cos\theta)^3 = 27\cos^3\theta$$

$$I = \int_0^{\pi/4} \frac{\frac{\cancel{9}}{4}\sin^2\theta}{\cancel{27}\cos^3\theta}\, \frac{\cancel{3}}{2}\cos\theta\, d\theta$$

$$= \frac{1}{8}\int_0^{\pi/4} \frac{\sin^2\theta}{\cos^2\theta}\, d\theta$$

$$= \frac{1}{8}\int_0^{\pi/4} \tan^2\theta \, d\theta$$

$$= \frac{1}{8}\int_0^{\pi/4} (\sec^2\theta - 1) \, d\theta$$

$$= \frac{1}{8}\Big[\tan\theta - \theta \Big]_0^{\pi/4}$$

$$= \frac{1}{8}\Big(\underbrace{\tan\frac{\pi}{4}}_{=1} - \frac{\pi}{4} - \underbrace{\tan 0}_{=0} + \underbrace{0}_{=0} \Big)$$

$$= \frac{1}{8}\left(1 - \frac{\pi}{4}\right)$$

$$= \frac{1}{8} - \frac{\pi}{32}$$

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$$I = \int \frac{x}{\sqrt{3-2x-x^2}}\,dx$$

COMPLETING THE SQUARE

$$3-2x-x^2 = 3-(x^2+2x)$$

$$=(3+1)-(x^2+2x+1) = 2^2-(x+1)^2$$

$$x+1 = 2\sin\theta \qquad x = 2\sin\theta - 1$$

$$dx = 2\cos\theta\,d\theta$$

$$I = \int \frac{2\sin\theta - 1}{\sqrt{2^2-2^2\sin^2\theta}}\,2\cos\theta\,d\theta$$

$$= \int \frac{2\sin\theta - 1}{\cancel{2}\cancel{\sqrt{1-\sin^2\theta}}}\,\cancel{2}\cancel{\cos\theta}\,d\theta$$

$$\sqrt{1-\sin^2\theta} = \cos\theta$$

$$= \int (2\sin\theta - 1)\,d\theta$$

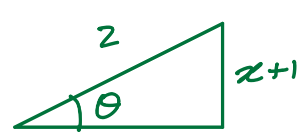
$$= -2\cos\theta - \theta + C$$

$$= -\sqrt{3-2x-x^2} - \arcsin\left(\frac{x+1}{2}\right) + C$$

$$x+1 = 2\sin\theta$$

$$\sin\theta = \frac{x+1}{2}$$

$$\theta = \arcsin\left(\frac{x+1}{2}\right)$$



$$\sqrt{2^2-(x+1)^2} = \sqrt{3-2x-x^2}$$