

MIDTERM 3 : APR 17

DETAILS SOON ON CANVAS

- OU INTEGRITY CODE
- OPEN BOOK/NOTES
- SELF TIMED (50 min)
- NO INTERNET • NO COMMUNICATION WITH OTHERS
- NO CALCULATORS THAT COMPUTE INTEGRALS.

2 TEST
RUNS

NEXT
WEEK

(LONG AS
2 QUIZZES)

$$\int_{1/3}^{\sqrt{2}/3} \frac{1}{x^5 \sqrt{9x^2 - 1}} dx$$

$$(x = \frac{1}{3} \sec \theta)$$

$$= \int_{1/3}^{\sqrt{2}/3} \frac{1}{x^5 \sqrt{(3x)^2 - 1}} dx$$

$$\begin{cases} 3x = \sec \theta \\ 3dx = \sec \theta \tan \theta d\theta \end{cases}$$

$$\bullet x = \frac{\sqrt{2}}{3}$$

$$\sec \theta = 3 \cdot \frac{\sqrt{2}}{3} = \sqrt{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\underline{\underline{\theta = \frac{\pi}{4}}}$$

$$\bullet x = \frac{1}{3} \Rightarrow \sec \theta = 1 \Rightarrow \underline{\underline{\theta = 0}}$$

$$= \int_0^{\pi/4} \frac{\left(\frac{1}{3}\right) \cancel{\sec \theta} \cancel{\tan \theta}}{\left(\frac{\sec \theta}{3}\right)^5 \sqrt{\cancel{\sec^2 \theta} - 1}} d\theta$$

$$= 3^4 \int_0^{\pi/4} \frac{1}{\sec^4 \theta} d\theta$$

$$= 3^4 \int_0^{\pi/4} (\cos^2 \theta)^2 d\theta = 3^4 \int_0^{\pi/4} \left(\frac{1}{2}(1 + \cos 2\theta)\right)^2 d\theta$$

$$= \frac{3^4}{2^2} \int_0^{\pi/4} (1 + 2 \cos 2\theta + \cos^2(2\theta)) d\theta$$

$$= \frac{3^4}{2^2} \int_0^{\pi/4} \left(\underbrace{1 + 2 \cos 2\theta + \frac{1}{2}(1 + \cos 4\theta)}_{\substack{\text{4/2} \\ \text{3/2}}} \right) d\theta$$

$$= \frac{3^4}{2^3} \int_0^{\pi/4} (3 + 4 \cos 2\theta + \cos 4\theta) d\theta$$

$$= \frac{3^4}{2^3} \left[3\theta + 2 \sin 2\theta + \frac{1}{4} \sin 4\theta \right]_0^{\pi/4}$$

$$= \frac{3^4}{2^3} \left[\left(3 \frac{\pi}{4} + 2 \sin \frac{\pi}{2} + \frac{1}{4} \sin \pi \right) - \left(\underset{\substack{'' \\ 0}}{0} + 2 \underset{\substack{'' \\ 0}}{\sin 0} + \frac{1}{4} \underset{\substack{'' \\ 0}}{\sin 0} \right) \right]$$

$$= \frac{3^4}{2^3} \left(3 \frac{\pi}{4} + 2 \right) = \frac{3^4}{2^5} (3\pi + 8)$$