

RATIONAL FUNCTION: $\frac{P(x)}{Q(x)}$

* PROPER: $\deg P(x) < \deg Q(x)$

DECOMPOSE INTO PARTIAL FRACTIONS

* IMPROPER: $\deg P(x) \geq \deg Q(x)$

LONG DIVISION BEFORE PARTIAL FRAC.

Example:

$$\int \frac{x^3 + x}{x-1} dx \quad \begin{array}{ll} P(x) = x^3 + x & \text{degree 3} \\ Q(x) = x-1 & \text{degree 1} \end{array}$$

IMPROPER

$$\begin{array}{r} x^2 + x + 2 \\ x-1 \overline{) \begin{array}{r} x^3 + + x \\ x^3 - x^2 \\ \hline x^2 + x \\ x^2 - x \\ \hline 2x \\ 2x - 2 \\ \hline 2 \end{array}} \end{array}$$

$$\frac{x^3 + x}{x-1} = x^2 + x + 2 + \frac{2}{x-1}$$

$$\int \frac{x^3 + x}{x-1} dx$$

$$= \int \left(x^2 + x + 2 + \frac{2}{x-1} \right) dx$$

$$= \frac{1}{3} x^3 + \frac{1}{2} x^2 + 2x + 2 \ln |x-1| + C$$

$$\int \frac{2x+1}{4x^2-4x+5} dx$$

COMPLETE
THE SQUARE

$$= \int \frac{2x+1}{(\underbrace{4x^2-4x+1}_{(2x-1)^2}) + 4} dx$$

$$= \int \frac{2x+1}{(2x-1)^2 + 4} dx \quad \left| \begin{array}{l} u = 2x-1 \\ du = 2 dx \\ 2x = u+1 \end{array} \right.$$

$$= \frac{1}{2} \int \frac{u+2}{u^2+4} du$$

$$= \frac{1}{4} \int \frac{2u}{u^2+4} du + \int \frac{1}{u^2+4} du$$

$$= \frac{1}{4} \ln |u^2+4| + \frac{1}{2} \arctan\left(\frac{u}{2}\right) + C \quad \left| u=2x-1 \right.$$

$$= \frac{1}{4} \ln |(2x-1)^2+4| + \frac{1}{2} \arctan\left(\frac{2x-1}{2}\right) + C$$
