$$\int \frac{1}{x^2 - x^4}\,dx$$

$$\frac{1}{x^2 - x^4} = \frac{1}{x^2(1-x^2)} = \frac{1}{x^2(1-x)(1+x)}$$

$$= \frac{x(1-x)(1+x)}{x(1-x)(1+x)} \cdot \frac{A}{x} = \frac{Ax(1-x)(1+x)}{x^2(1-x)(1+x)}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1-x} + \frac{D}{1+x}$$

$$= \frac{Ax(1-x)(1+x) + B(1-x)(1+x) + Cx^2(1+x) + Dx^2(1-x)}{x^2(1-x)(1+x)}$$

$$\begin{aligned} 1 = \; & A\,x(1-x)(1+x) \\ & + B\,(1-x)(1+x) \\ & + C\,x^2(1+x) \\ & + D\,x^2(1-x) \end{aligned}$$

$\underline{x=-1}$ $\quad 1 = D\cdot(-1)^2(1-(-1))$ $\quad \boxed{D = \frac{1}{2}}$

$\underline{x=1}$ $\quad 1 = C\,1^2(1+1)$ $\quad \boxed{C = \frac{1}{2}}$

$\underline{x=0}$ $\quad 1 = B(1-0)(1+0)$ $\quad \boxed{B = 1}$

$\underline{x=2}$ $\quad 1 = A(-6) + B(-3) + C\,12 + D(-8)$

$$1 = -6A - 3 + 6 - 4 = -6A - 1$$

$$2 = -6A \qquad \boxed{A = -\frac{1}{3}}$$

$$\int \left(\frac{-1/3}{x} + \frac{1}{x^2} + \frac{1/2}{1-x} + \frac{1/2}{1+x} \right) dx$$

$$= -\frac{1}{3} \ln|x| - \frac{1}{x} - \frac{1}{2} \ln|1-x| + \frac{1}{2} \ln|1+x| + C$$

$$\int \frac{x^5 + x^3 + 2x^2 + 1}{x^4 + x^2} \, dx \quad = I$$

$$\begin{array}{r|l} & \quad x \\ \hline x^4 + x^2 & \quad x^5 + x^3 + 2x^2 + 1 \\ & \quad x^5 + x^3 \\ \hline & \qquad\qquad \underbrace{2x^2 + 1} \end{array}$$

$$\frac{x^5 + x^3 + 2x^2 + 1}{x^4 + x^2} = x + \frac{2x^2 + 1}{x^4 + x^2} = x + \frac{2x^2 + 1}{x^2 \underbrace{(x^2+1)}_{\text{IRREDUCIBLE QUADRATIC}}}$$

$$\frac{2x^2 + 1}{x^2 (x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$$

$$2x^2 + 1 = A\,x\,(x^2+1) + B\,(x^2+1) + (Cx + D)\,x^2$$

$$\frac{2x^2+1}{x^2(x^2+1)} = \frac{(x^2)+(x^2+1)}{x^2\,(x^2+1)} =$$

$$= \frac{\cancel{x^2}}{\cancel{x^2}(x^2+1)} + \frac{\cancel{x^2+1}}{x^2\cancel{(x^2+1)}}$$

$$= \frac{1}{x^2+1} + \frac{1}{x^2}$$

$A = 0$
$B = 1$
$C = 0$
$D = 1$

$$I = \int \left(x + \frac{1}{x^2+1} + \frac{1}{x^2} \right) dx$$

$$= \frac{1}{2}x^2 + \arctan(x) - \frac{1}{x} + C$$

$$\int \frac{x^4 - 3x^3 + 4x^2 - 2x + 1}{x^2 - 2x + 2}\, dx$$

TRY THIS
AT HOME

$$I_2 = \int \frac{1}{\underbrace{x^2 - 2x + 2}}\, dx$$

IRRED.
QUAD.

COMPLETE
THE SQUARE

$$I_2 = \int \frac{1}{\underbrace{(x^2 - 2x + 1)}_{= (x-1)^2} + 1} \, dx$$

$$= \int \frac{1}{(x-1)^2 + 1} \, dx$$

$u = x - 1$

$du = dx$

$$= \int \frac{1}{u^2 + 1} \, du$$

$$= \arctan(u) + C$$

$$= \arctan(x-1) + C$$