$$\int \frac{1}{x + x\sqrt{x}}\,dx \qquad \begin{aligned} u &= \sqrt{x} \\ u^2 &= x \\ 2u\,du &= dx \end{aligned}$$

$$= \int \frac{2u}{u^2 + u^2 u}\,du = \int \frac{2}{u + u^2}\,du$$

$$= \int \frac{2}{u(1+u)}\,du$$

$$\frac{2}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u}$$

$$2 = A(1+u) + Bu$$

$u = 0$ $\quad \boxed{A = 2}$

$u = -1$ $\quad \boxed{B = -2}$

$$= \int \frac{2}{u}\,du - \int \frac{2}{1+u}\,du$$

$$= 2\ln|u| - 2\ln|1+u| + C$$

$$= 2\ln\sqrt{x} - 2\ln(1+\sqrt{x}) + C$$

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^3}\,dx \qquad \begin{aligned} u &= \frac{1}{x} \\ du &= -\frac{1}{x^2}\,dx \end{aligned}$$

$$= \int \underbrace{\cos\left(\frac{1}{x}\right)}_{\cos u} \underbrace{\frac{-1}{x}}_{-u} \cdot \underbrace{\frac{-1}{x^2}\,dx}_{du}$$

$$= -\int (\cos u)\, u\, du \qquad \text{FINISH BY PARTS}$$

$$\int \sqrt{1-\sin x}\; dx$$

$$= \int \sqrt{1-\sin x}\, \frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}}\, dx$$

$\cos^2 x + \sin^2 x = 1$

$1 - \sin^2 x = \cos^2 x$

$\sqrt{1-\sin^2 x} = \cos x$

~~DOES NOT WORK~~

$$= \int \frac{\sqrt{(1-\sin x)(1+\sin x)}}{\sqrt{1+\sin x}}\, dx$$

$$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1+\sin x}}\, dx = \int \frac{\cos x}{\sqrt{1+\sin x}}\, dx$$

$$= \int \frac{1}{\sqrt{u}}\, du$$

$u = 1 + \sin x$

$du = \cos x \, dx$

$$= 2\sqrt{u} + C$$

$$= 2\sqrt{1+\sin x} + C$$

$$\int e^{x+e^x}\, dx$$

$u = x + e^x$

$du = (1+e^x)\,dx$

$u = e^x$

$du = e^x\, dx$

$$= \int \overbrace{e^{x}\cdot \underbrace{e^{e^{x}}}_{e^u}\, dx}^{du} = \int e^u\, du = e^u + C = e^{e^x} + C$$

$$\int x\underbrace{\sqrt{2-\sqrt{1-x^2}}}_{u}\,dx$$

$x\,dx = \ldots\, du$

$$u = \sqrt{2-\sqrt{1-x^2}}$$

$$u^2 = 2-\sqrt{1-x^2}$$

$$\sqrt{1-x^2} = 2-u^2$$

$$1-x^2 = (2-u^2)^2 = 4-4u^2+u^4$$

$$-2x\,dx = (-8u+4u^3)\,du$$

$$x\,dx = (4u-2u^3)\,du$$

$$= \int u\,(4u-2u^3)\,du$$

$$= \int (4u^2-2u^4)\,du = \frac{4}{3}u^3 - \frac{2}{5}u^5 + C$$

$$= \frac{4}{3}\left(\sqrt{2-\sqrt{1-x^2}}\right)^3 - \frac{2}{5}\left(\sqrt{2-\sqrt{1-x^2}}\right)^5 + C$$