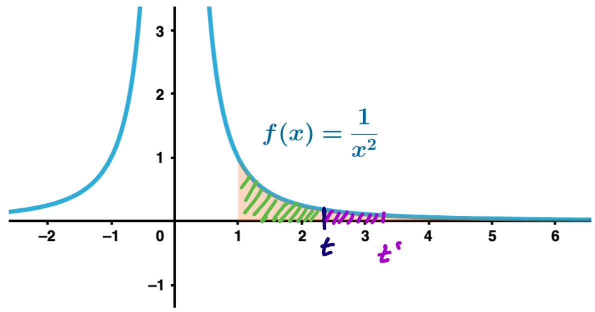
$$\int_1^\infty \frac{1}{x^p}\,dx$$

$p = 2$



$p \neq 1$

$$\int_1^\infty \frac{1}{x^p}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx$$

$$= \lim_{t\to\infty} \int_1^t x^{-p}\,dx = \lim_{t\to\infty} \left[\frac{x^{-p+1}}{-p+1}\right]_1^t$$

$$= \lim_{t\to\infty} \left(\frac{t^{-p+1}}{-p+1} - \frac{1}{-p+1}\right) = (*)$$

Two cases: $\underline{p > 1}$

$$(*) = \lim_{t\to\infty} \left(\frac{1}{-p+1}\Big(\underbrace{\frac{1}{t^{p-1}}}_{\xrightarrow{p>1} 0} - 1\Big)\right)$$

$$= \left(\frac{1}{-p+1}(0-1)\right) = \frac{1}{p-1} \quad \text{CONVERGENT}$$

$(*)$ $p < 1$

$$(*) = \lim_{t\to\infty} \left(\frac{1}{-p+1}\Big(\underbrace{t^{-p+1}}_{\xrightarrow{p<1} +\infty} - 1\Big)\right) = +\infty$$

DIVERGENT

$\boxed{2}$ $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ is convergent if $p > 1$ and divergent if $p \leqslant 1$.

$$\int_{-\infty}^{0} x e^x \, dx = \lim_{t \to -\infty} \underbrace{\int_t^0 x e^x \, dx} = (*)$$

$$\int \underbrace{x}_{u} \, \underbrace{e^x \, dx}_{dv} = x e^x - \int e^x \, dx = x e^x - e^x + C$$

$du = dx$

$v = e^x$

$$(*) = \lim_{t \to -\infty} \Big[x e^x - e^x \Big]_t^0 = \lim_{t \to -\infty} \Big[\big(\underbrace{0 \cdot e^0}_{0} - \underbrace{e^0}_{1} \big) - \big(t e^t - e^t \big) \Big]$$

$$= \lim_{t \to -\infty} \big(\underbrace{e^t}_{\to 0} \underbrace{- t}_{\to \infty} \underbrace{e^t}_{\to 0} - 1 \big) = 0 + 0 - 1 = -1$$

$\infty \cdot 0$ INDETERMINATE !

$$\lim_{t \to -\infty} \left(-t e^t \right) = \lim_{t \to -\infty} \frac{-t}{e^{-t}} \overset{\frac{\infty}{\infty}\ L'H}{=} \lim_{t \to -\infty} \frac{-1}{-e^{-t}} = 0$$

TYPE $\infty \cdot 0$

$-t \to \infty$, $e^{-t} \to \infty$

$$\int_{-\infty}^{\infty} \sin^2 x \, dx$$ SPLIT THE INTEGRAL!

$$= \int_{-\infty}^{0} \sin^2 x \, dx + \int_{0}^{\infty} \sin^2 x \, dx$$

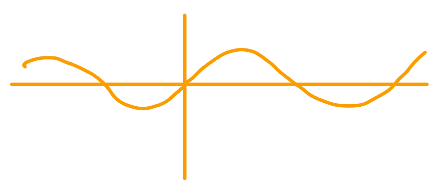
$$\int_{0}^{\infty} \sin^2 x \, dx = \lim_{t \to \infty} \int_{0}^{t} \sin^2 x \, dx$$

$$= \lim_{t \to \infty} \int_{0}^{t} \frac{1}{2}(1 - \cos 2x) \, dx$$

$$= \lim_{t \to \infty} \left[\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) \right]_{0}^{t}$$

$$= \lim_{t \to \infty} \left[\frac{1}{2}\left(t - \frac{1}{2}\sin 2t\right) - \frac{1}{2}\left(0 - \frac{1}{2}\underbrace{\sin 0}_{=0}\right) \right]$$

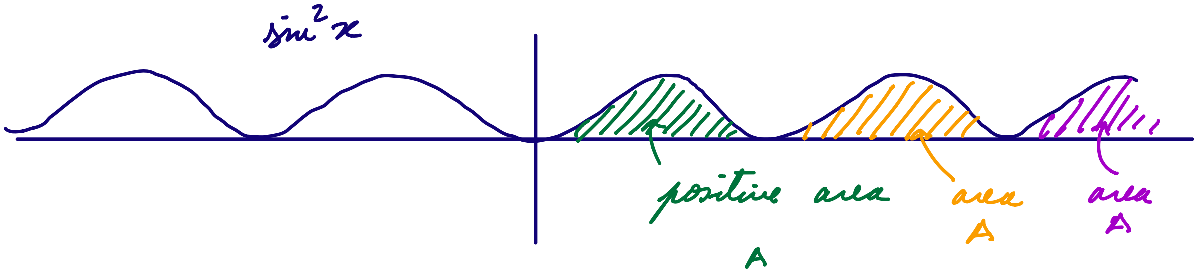
$$= \lim_{t \to \infty} \frac{1}{2}\left(\underbrace{t}_{\to \infty} - \frac{1}{2}\underbrace{\sin 2t}_{\text{DNE, bounded in } [-1,1]}\right)$$



$$= \infty$$

Hence: $\int_{0}^{\infty} \sin^2 x \, dx = \infty$ DIVERGENT

Hence: $\int_{-\infty}^{\infty} \sin^2 x \, dx$ DIVERGENT



$$\int_0^\infty \sin^2 x \, dx = A + A + A + A + \cdots$$