Comparison Theorem Suppose that f and g are continuous functions with $f(x) \geqslant g(x) \geqslant 0$ for $x \geqslant a$.
(a) If $\int_a^\infty f(x)\,dx$ is convergent, then $\int_a^\infty g(x)\,dx$ is convergent.
(b) If $\int_a^\infty g(x)\,dx$ is divergent, then $\int_a^\infty f(x)\,dx$ is divergent.

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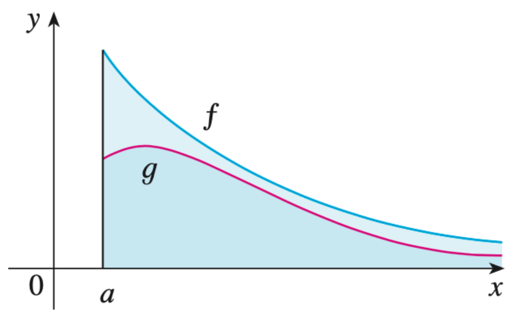
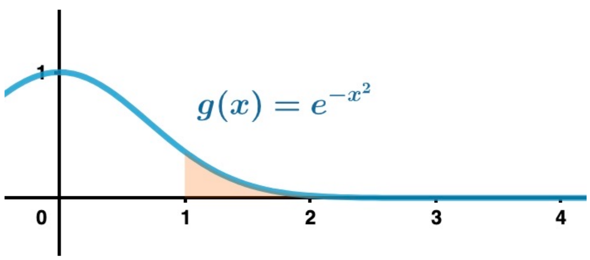
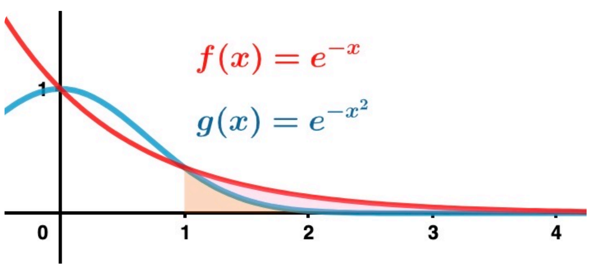




$$\int_1^\infty e^{-x^2}\,dx$$ CONVERGENT or DIVERGENT ?

PROBLEM: CAN'T INTEGRATE



IDEA: COMPARE TO:

$$\int_1^\infty e^{-x}\,dx$$

① $x \geqslant 1 \Rightarrow x^2 \geqslant x \Rightarrow -x^2 \leqslant -x \Rightarrow e^{-x^2} \leqslant e^{-x}$

② $\displaystyle\int_1^\infty e^{-x}\,dx = \lim_{t\to\infty} \int_1^t e^{-x}\,dx$

$\displaystyle = \lim_{t\to\infty} \left[-e^{-x}\right]_1^t = \lim_{t\to\infty} \left(-e^{-t} + e^{-1}\right) = 0 + e^{-1} = e^{-1}$

$$\int_1^\infty e^{-x}\,dx \quad \text{CONVERGENT}$$

$$0 \leq e^{-x^2} \leq e^{-x} \quad \text{for } x \geq 1$$

COMPARISON THEOREM

$$\Rightarrow \int_1^\infty e^{-x^2}\,dx$$

CONVERGENT