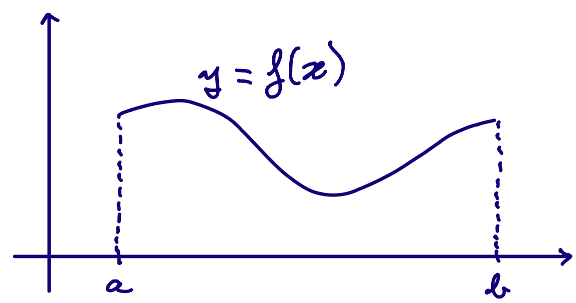
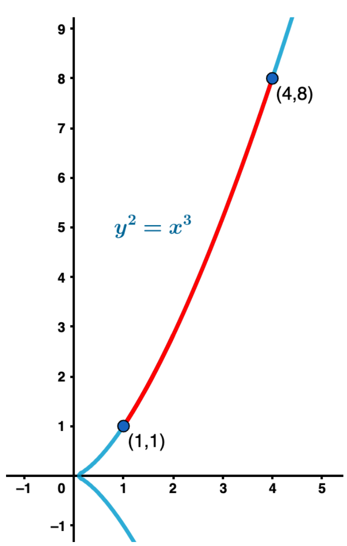
2 **The Arc Length Formula** If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leqslant x \leqslant b$, is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

L = LENGTH of the curve





$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$y^2 = x^3$ $\qquad$ $y = x^{3/2}$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

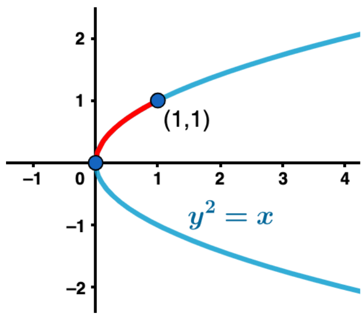
$$L = \int_1^4 \sqrt{1 + \left(\frac{3}{2} x^{1/2}\right)^2}\, dx$$

$$= \int_1^4 \sqrt{1 + \frac{9}{4} x}\, dx$$

$$= \ldots\ldots = \frac{1}{27}\left(8\sqrt{10} - 13\sqrt{13}\right)$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

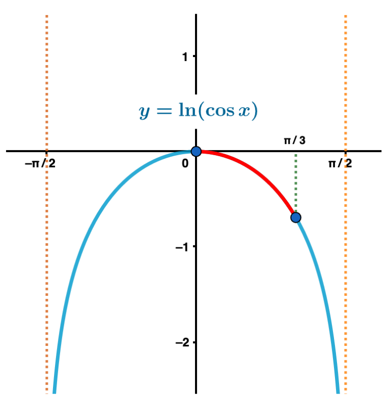
$$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$



$x = y^2 \qquad \frac{dx}{dy} = 2y$

$$L = \int_0^1 \sqrt{1 + 4y^2}\, dy$$

$$= \ldots\ldots = \frac{\sqrt{5}}{2} + \frac{\ln(2+\sqrt{5})}{4}$$



$y = \ln(\cos x)$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x$$

$$L = \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx =$$

$$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/3} \sec x\, dx$$

$$= \Big[\ln|\sec x + \tan x|\Big]_0^{\pi/3} = \underline{\ln|2 + \sqrt{3}|}$$