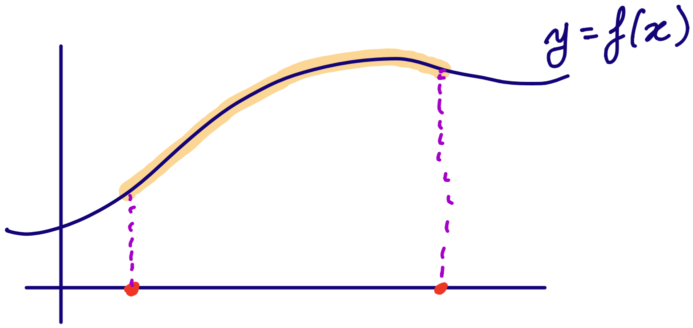
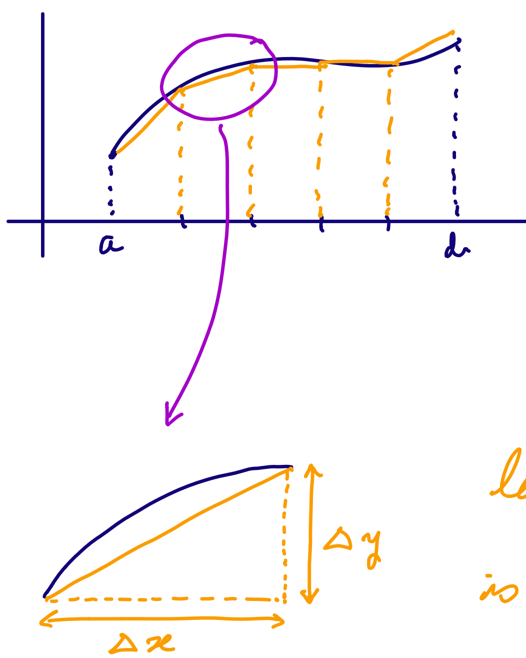
$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

ACR LENGTH





approximate L by the sum of the lengths of the segments

length of the segment is : $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

Note : $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$

Length is approx: $\displaystyle\sum_{i=1}^{n} \sqrt{1+\left(\frac{\Delta y_i}{\Delta x}\right)^2}\ \Delta x$

(for the area, it would be $\displaystyle\sum_{i=1}^{n} f(x_i^*)\,\Delta x$)

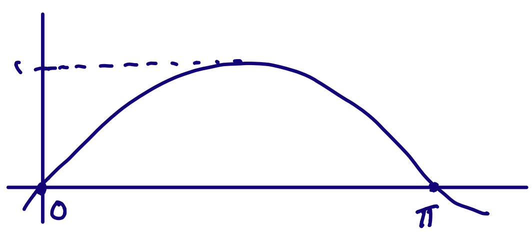
Taking limits:

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\boxed{L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}$$

$$\boxed{L = \int_c^d \sqrt{1+[g'(y)]^2}\,dy = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy}$$

Ex: $y = \sin x \qquad 0 \leq x \leq \pi$



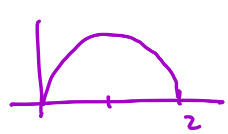
$$\frac{dy}{dx} = \cos x \qquad \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\cos^2 x}$$

$$L = \int_0^{\pi} \sqrt{1+\cos^2 x}\,dx$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

Note: half circle: $\sqrt{1-(x-1)^2} \neq \sin x$



SET UP INTEGRALS COMPUTING ARC LENGTH:

① $y = \ln(\sec x) \qquad 0 \le x \le \pi/4$

② $y = x e^{-x} \qquad 0 \le x \le 2$

③ $x + 2y = y^2 \qquad 0 \le y \le 2$

④ $y^2 = \ln x \qquad -1 \le y \le 1$

① $\dfrac{dy}{dx} = \dfrac{\cancel{\sec x}\tan x}{\cancel{\sec x}} = \tan x$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/4} \sec x \, dx$$

② $\dfrac{dy}{dx} = e^{-x} - x e^{-x} \qquad L = \displaystyle\int_0^2 \sqrt{1 + (e^{-x} - xe^{-x})^2}\, dx$

③ $x = y^2 - 2y$

$\dfrac{dx}{dy} = 2y - 2 \qquad L = \displaystyle\int_0^2 \sqrt{1 + (2y-2)^2}\, dy$

④ $y^2 = \ln x \qquad \dfrac{dx}{dy} = e^{y^2} \cdot 2y$

$x = e^{y^2}$

$y = \sqrt{\ln x} \qquad L = \displaystyle\int_{-1}^{1} \sqrt{1 + (e^{y^2} 2y)^2}\, dy$

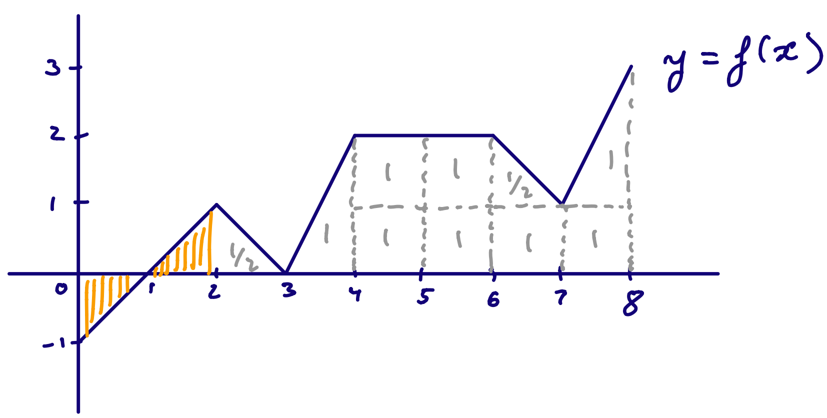
$$y^2 = \ln x \xrightarrow[\frac{d}{dy}]{} 2y = \frac{1}{x}\frac{dx}{dy}$$

$$\frac{dx}{dy} = x \cdot 2y = e^{y^2} 2y$$

AVERAGE VALUE

f defined on $[a, b]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$



$$f_{ave} = \frac{1}{8-0}\int_0^8 f(x)\,dx = \frac{1}{8}\cdot 9 = \frac{9}{8}$$

in $[0, 8]$