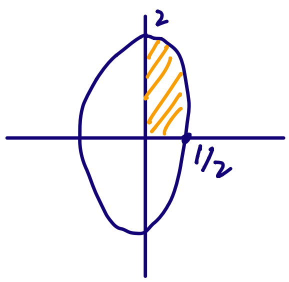
$16x^2 + y^2 = 4$



$$A = 4\int_0^{1/2} \cdots \, dx$$

$$16x^2 + y^2 = 4$$

$$y^2 = 4 - 16x^2$$

$$y = \pm\sqrt{4 - 16x^2}$$

$$A = 4\int_0^{1/2} \sqrt{4 - 16x^2}\, dx$$

$$= 8\int_0^{1/2} \sqrt{1 - (2x)^2}\, dx$$

$2x = \sin\theta$

$2\,dx = \cos\theta\, d\theta$

$x = 0 \rightarrow \theta = 0$

$x = 1/2 \rightarrow \theta = \frac{\pi}{2}$

$$= 4\int_0^{\pi/2} \sqrt{1 - \sin^2\theta}\, \cos\theta\, d\theta$$

$$= 4\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 2\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$$

$$= 2\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = 2\left(\left(\frac{\pi}{2} + \frac{1}{2}\underbrace{\sin\pi}_{=0}\right) - \left(\underbrace{0}_{=0} + \frac{1}{2}\underbrace{\sin 0}_{=0}\right)\right)$$

$$= 2\cdot\frac{\pi}{2} = \pi$$

$$\int \frac{x^3-2x+5}{x^2-2x+2}\,dx = I$$

$$\begin{array}{r|l}
 & \quad x \;\; +2 \\
\hline
x^2-2x+2 & x^3 \qquad -2x \;\; +5 \\
 & -(x^3-2x^2+2x) \\
\hline
 & \qquad 2x^2-4x \;\; +5 \\
 & \quad -(2x \;\; -4x \;\; +4) \\
\hline
 & \qquad\qquad\qquad 1
\end{array}$$

$$I = \int \left(x+2 + \frac{1}{x^2-2x+2} \right) dx$$

$$x^2-2x+2 = (x^2-2x+1) + 1$$

$$= (x-1)^2+1$$

$$I = \int \left(x+2 + \frac{1}{(x-1)^2+1} \right) dx$$

$$= \frac{x^2}{2} + 2x + \arctan(x-1) + C$$

x	$f(x)$	$f'(x)$	$f''(x)$
0	3	8	25
1	0	-5	2

$$\int_0^1 \frac{1}{2} x^2 f'''(x)\, dx$$

$u = \frac{1}{2} x^2 \qquad dv = f'''(x)dx$

$du = x\, dx \qquad v = f''(x)$

$$= \left[\frac{1}{2} x^2 f''(x) \right]_0^1 - \int_0^1 x f''(x)\, dx$$

$u = x \qquad dv = f''(x)dx$

$du = dx$

$v = f'(x)$

$$= \left[\frac{1}{2} x^2 f''(x) - x f'(x) \right]_0^1 + \int_0^1 f'(x)\, dx$$

$$= \left[\frac{1}{2} x^2 f''(x) - x f'(x) + f(x) \right]_0^1$$

$$= \left(\frac{1}{2} f''(1) - f'(1) + f(1) \right) - \left(0 - 0 + f(0) \right)$$

$$= \frac{1}{2} 2 - (-5) + 0 - 3 = 4$$

$$I = \int \underbrace{x}_{\text{a polynomial}} \, \underbrace{\ln x}_{\text{easy to take derivatives}} \, dx \quad = $$

$u = \ln x \qquad dv = x\, dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{2}x^2$

$$= \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$u = x \qquad dv = \ln x \, dx$

$du = dx \qquad v = (x \ln x - x)$

$$I = x^2 \ln x - x^2 - \int (x \ln x - x)\, dx$$

$$= x^2 \ln x - x^2 + \int x\, dx - \underbrace{\int x \ln x \, dx}_{= I}$$

$$2I = x^2 \ln x - x^2 + \frac{1}{2} x^2 + C$$

$$= x^2 \ln x - \frac{1}{2} x^2 + C$$

$$\boxed{I = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C'}$$