15

Valuing Liquidity and Control

INTRODUCTION

The simple premise of much M&A valuation analysis is that the target shareholder sells total *control* in the target firm and that the securities or assets being acquired are marketable. This chapter explores the world beyond that premise. The vast majority of M&A deals involve privately owned firms and illiquid stock. Exhibit 15.1 shows the volume of acquisitions involving the purchase of a privately owned target, or the purchase of a minority interest in a company. From 1990 to 2002 acquisitions of minority interests accounted for between 1 and 3 percent of all transactions. During the same period, the purchase of independent entities that were private targets accounted for 40 to 60 percent of all deals. Divestitures by corporations represent the sale of illiquid securities or assets. Exhibit 6.15 in Chapter 6 presents corporate divestitures as a percentage of all M&A activity. From 1990 to 2002, divestitures averaged 33 percent of all transactions. In short, the need to apply illiquidity discounts and control premiums is more the rule than the exception in M&A.

The realm of illiquidity and lack of control is a focus of detailed analysis and lively debate. Shannon Pratt (2001, page 37) wrote:

> *There is often more money in dispute in determining the discounts and premiums in a business valuation than in arriving at the pre-discount valuation itself. Discounts and premiums affect not only the value of the company but also play a crucial role in determining the risk involved, control issues, marketability, contingent liability, and a host of other factors that can make or break a deal.*

This chapter summarizes what we know from research and offers a new framework for valuing *liquidity* and control based on the theory of real options. Learnings include:

- Illiquidity requires a discount from liquid values.
- Minority status requires a discount from value with 100 percent control.
- Liquidity and control are rights, and may be assessed in terms of their option value. This is a new way to think about these effects. This chapter summarizes recent research.

Acquisitions of Minority Interests - U.S.
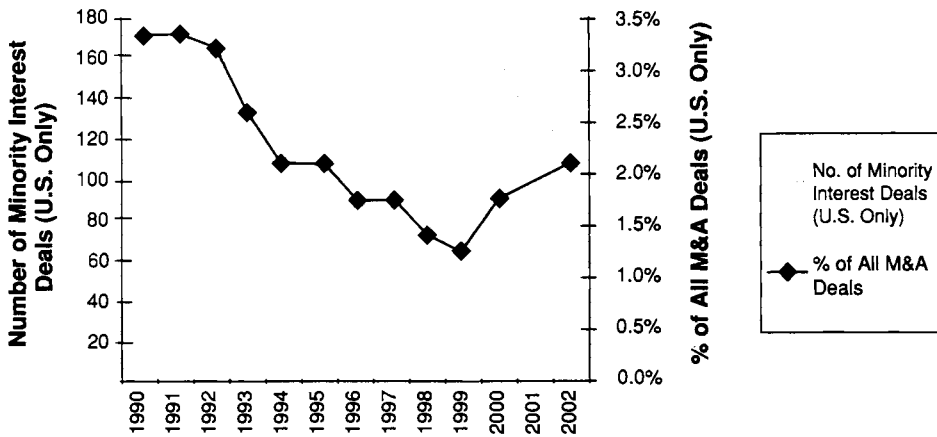
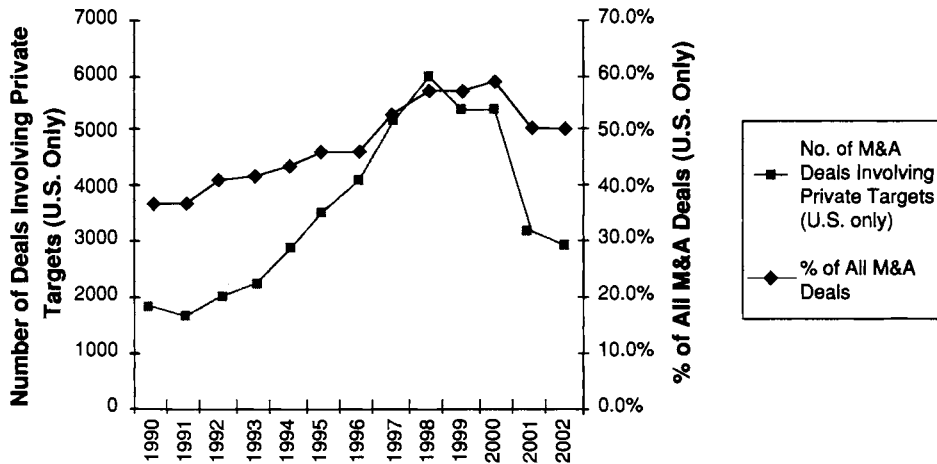**Acquisitions of Private Targets - U.S.**

EXHIBIT 15.1   M&A Activity Involving Acquisition of Minority Interests
Source of data: Thomson Financial SDC Database.

- The traditional approach is to assess discounts and premiums based on the analysis of peer deals. This chapter illustrates the calculations under this approach.
- Liquidity and control can have sizable effects on shareholder welfare.

ADJUSTING VALUES FOR DISCOUNTS AND PREMIUMS

The point of departure into the realm of liquidity and control is conventional practice. This section of the chapter surveys the traditional means of adjusting a purchase price for illiquidity and lack of control.

Discounts and Premiums Start from a Base

Whenever liquidity and/or control change, value changes. We can think of the value of the firm as a composite of the stand-alone value of the target plus a *discount or premium* for liquidity and control.

$$\text{Maximum payment for Target} = V_{\text{Stand-alone}} + V_{\text{Synergies}} + \Delta_{\text{Illiquidity and control}} \quad (1)$$

where: Δ is the change in value from the base case resulting from effects of illiquidity and control. This term could be positive or negative.

The "base case" valuation of the stand-alone firm comes from conventional valuation approaches, such as DCF and multiples (described in Chapter 9). These assume, in effect, that you buy a small interest in liquid shares of stock and that the firm continues to operate as is. Chapter 11 recommended valuation of the firm with synergies, to give a sense of the economic upside. The sum of stand-alone value and synergy value form the base case value of the firm.

Base case values estimated with DCF or multiples of earnings implicitly assume that the firm's shares are liquid and that all shareholders are governing; no block of shares retains special control rights over the firm. Deviations from these two assumptions must trigger adjustments in value—this is where the illiquidity discount and *control premium* come in. Exhibit 15.2 sketches an example of possible changes. The base case is in the southeast corner of the diagram, the firm with liquid shares and no *control asymmetries*—this means that there are no
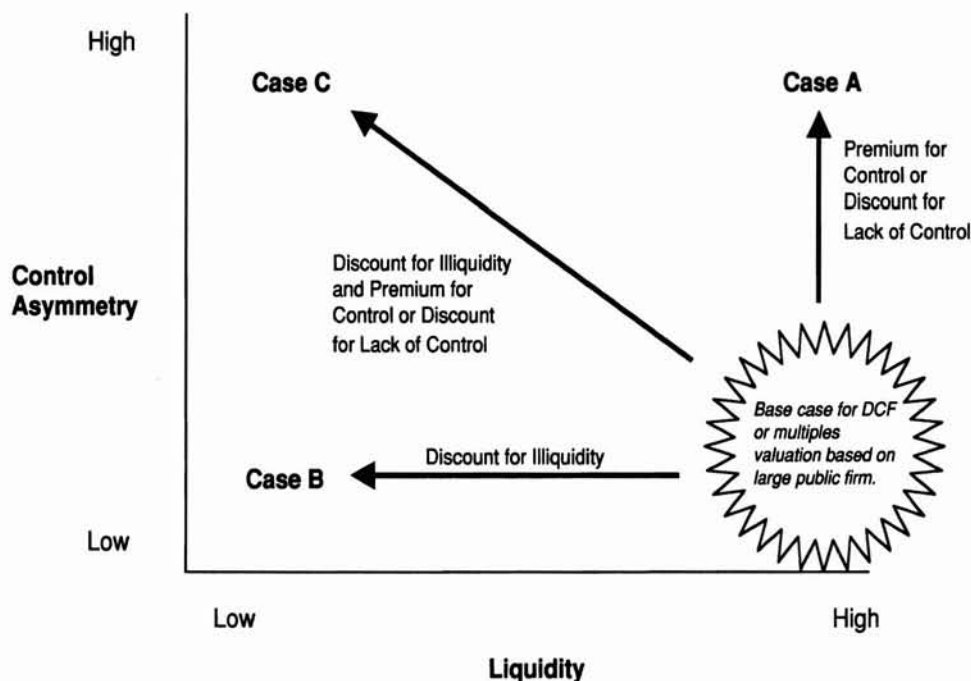

EXHIBIT 15.2 Deviations in Liquidity and Control from Base Case Require a Premium or Discount

groups of shareholders with special control rights. As you move from liquidity to illiquidity, the shareholder must sustain a discount in the value of his or her shares. And as you move from the base case of no control asymmetries to the world where control asymmetries exist, two things happen: The control group gains a premium value to their shares, while the minority shareholders experience a discount—the control group and minority group diverge in value as the control asymmetry grows.

All discussion of premiums and discounts begins with some base case. As a convention in the discussion that follows, the base case will be the value of the firm as if its shares were actively traded on a public exchange (i.e., liquid, and therefore marketable) and as if there were no shareholders with an unusual degree of power over the strategic decisions of the firm—General Electric is a good example of the firm with these qualities. We could just as easily choose a different base case, which would cause us to alter our use of "premium" and "discount" in the discussion of liquidity and control effects.

By the way, premiums and discounts are inversely related. You can convert from one to the other with this formula:

$$\% \text{ discount} = 1 - \left(\frac{1}{1 + \% \text{ premium}} \right) \tag{2}$$

The Multiplicative Model—the Traditional Approach

Liquidity and control are valuable. The practitioner must adjust the payment in an acquisition in line with equation (1). The mechanics of this have been sharpened in practice. Pratt (2001) advocates adjusting the total estimated value of the firm using discounts and premiums in a multiplicative[1] model.[2] In the multiplicative model, the effects of liquidity and control are compounded:

$$\text{Gross base price}(1 + \pi_{\text{Control}})(1 - \delta_{\text{Illiquidity}}) = \text{Net price} \tag{3}$$

where π = Premium for control group over value existing in base case. This would be a negative value for minority group.

δ = Discount for illiquidity that may exist in comparison to value in base case. Illiquidity always is a negative effect.

Example of the Traditional Approach: Three Prospective Acquisitions

To illustrate the calculations, consider the following problem. You are the CEO of a firm listed on the New York Stock Exchange (NYSE); your shareholders are widely dispersed, with no shareholder having more controlling influence than the others. You are considering making three acquisitions. Each firm has a DCF value of $100 million and has 100 million shares outstanding. The "base" from which any adjustments should be made is a share price of $1.00. You believe the DCF value was derived in a way that assumes liquid shares and no control asymmetry. Thus, illiquidity will impose a discount from the base value and control will command a premium relative to the base case.[3]

- *Target A: publicly held company with a majority shareholder.* One shareholder owns 51 percent of the shares of this company. You wonder what share price to offer the majority shareholder, and what price to offer the minority shareholders. Privately, you believe a 40 percent control premium over the base case valuation is justified.
- *Target B: privately held company with dispersed shareholders.* Here there is no control asymmetry arising from the existence of a control block and of a group of minority shareholders. But the shares to be purchased are illiquid. Privately you believe a 30 percent discount from the base case for illiquidity is warranted.
- *Target C: privately held company with a majority shareholder.* One shareholder owns 51 percent of the shares of this company. All shares are illiquid. You wonder what share price to offer the majority shareholder, and what price to offer the minority shareholders. Privately, you believe a 40 percent control premium and a 30 percent illiquidity discount are justified, relative to the base case valuation.

Note that a control asymmetry will grant a premium to the controlling block and impose a discount on the minority block. Based on the proportions of share ownership, you would be justified in quoting different prices for the control and minority shares.

Exhibit 15.3 computes the adjusted values, showing the impact of the liquidity discount and control premium. This exhibit is drawn from "Liquidity and Control.xls" that is found on the CD-ROM. Exhibit 15.4 summarizes the share price results of the cases in a two-way matrix for liquidity and control that map onto the qualitative presentation in Exhibit 15.2. In the southeast corner is the base case, where all shares command a price of $1.00. In the southwest corner, the shares are subject to a liquidity discount only; in the absence of a control asymmetry all shares command a price of $0.70, reflecting the 30 percent liquidity discount.

The top half of Exhibit 15.4 explores the impact of a control asymmetry. In the northeast corner, shares are liquid, but there is a control asymmetry that causes the control and minority shares to be valued differently. The control block shares are worth $1.40 (reflecting the 40 percent control premium you assumed). And the minority shares are worth $0.58—the majority's gain is the minority's loss! The minority may feel that this discount represents an expropriation of their value. The majority no doubt feels that the premium justly compensates them for the rights of control. But under the conservation of value (one of the deep principles in finance, which can be found in the writings of Modigliani and Miller) the net effect of the control asymmetry must sum to zero.

Finally, in the northwest corner of the table, we have the world of illiquidity and control asymmetry. Here, the control block shares are worth $0.98—barely different from the base case ($1.00) because of the offsetting effects of illiquidity and control. For minority shareholders, however, shares are worth only $0.41: a dramatic discount from the base case and the worst case of all.

This example demonstrates that the impact of liquidity discounts and control premiums on share prices can be dramatic. Also, relatively small changes in the premium and discount can produce material swings in the adjusted share prices.

EXHIBIT 15.3   Three Example of the Multiplicative Approach for Estimating Illiquidity and Control Effects on Value

Assumptions				
% discount for illiquidity			30%	
% premium for control			40%	
Size of control block			51%	
Base case value of the equity; marketable, no control			$100	
Number of shares outstanding			100	

	Case A	Case B	Case C	Note
Illiquidity? ("Yes" if shares are not liquid)	No	Yes	Yes	
Control asymmetry? ("Yes" if there is a control block)	Yes	No	Yes	
1 Base case value of the equity; marketable, no control asymmetries	$100.00	$100.00	$100.00	A
2 Adjustment for illiquidity	0%	−30%	−30%	B
3 Value of equity adjusted for possible illiquidity	$100.00	$ 70.00	$ 70.00	B
4 % premium for control	40%	0%	40%	C
5 Size of controlling block	51%	0%	51%	D
6 Value of controlling block	$ 71.40	$ —	$ 49.98	E
7 Value of minority block	$ 28.60	$ 70.00	$ 20.02	F
8 Value of equity adjusted for control asymmetry and illiquidity	$100.00	$ 70.00	$ 70.00	G
9 Controlling block price per share	$ 1.40	N/A	$ 0.98	
10 Minority block price per share	$ 0.58	N/A	$ 0.41	
11 Price to all if no control asymmetry	N/A	$ 0.70	N/A	

Notes:
A. Start with the value of the firm with marketable shares but no control blocks—like large, publicly traded corporations. This is the base case from which most valuation adjustment approaches begin.
B. Illiquidity is assumed to affect *all* shares equally. Therefore, the first adjustment must be for illiquidity.
C. The presence of a control block affects the control group and minority differently. Therefore, control must be addressed after liquidity.
D. The size of the control block determines what portion of the equity will receive the control premium.
E. The value of the control block equals the value of equity adjusted for illiquidity (line 3) times (1 + Control premium) (line 4) times % size of control block (line 5).
F. The value of the minority block equals the difference between the value of equity adjusted for possible illiquidity (line 3) and the value of the control block (line 6).
G. The value of minority and control blocks to sum to the value of equity adjusted for possible illiquidity.

Analysts obtain these parameters by studying other M&A transactions that are comparable in terms of size, industry, and other factors. Specialist consultants maintain proprietary databases for the purpose of generating suggested premiums and discounts. But the analysis of comparable transactions leaves considerable room for judgment. For instance, one analyst wrote:

It would seem at first glance that control premiums paid in buyouts of public companies would be ideal indicators of the magnitude of discount necessary for proper valuation of a minority interest. Yet it becomes apparent that such data is compiled from such a diverse field that its usefulness is limited. This diversity is caused by differences in the degree of control obtained, the industry of the acquired company, the timing of the buyout, the concentration of control among selling shareholders, the perceived benefits or synergies to be obtained by buyers, the receptiveness of management to the offer, and the presence or absence of competitive bids. Finding enough examples from which to draw a valid discount conclusion for a specific degree of control in a specific industry during a given time period is rarely, if ever, possible. (Pratt 2001, page 20)

This is a strong and telling statement, which suggests that comparable transactions analysis is helpful, but does not provide definitive answers.

The analyst needs a principled basis for his or her recommendations. Where do discounts and premiums come from? What factors drive them? Does one size fit all cases? And is there any test of reasonableness? The next three sections offer some insights drawn from financial economics, including these:

■ Discounts and premiums arise from the optionality embedded in liquidity and control.
■ Uncertainty about the value of the target is a significant driver of discounts and premiums. Also, because liquidity and control rights are driven by the same underlying factors, the two options *interact* on the value of each other. Liquidity and control effects are not independent.
■ One size does not fit all. Deal makers should not impose a discount or premium in fixed fashion across all transactions. Rules of thumb are likely to be inappropriate.

EXHIBIT 15.4 Summary of Three Case Examples: Offered Share Price as It Varies with Assumptions about Illiquidity and Control Asymmetry

Control	Liquid Shares	
Asymmetry	No	Yes
Yes: Control	$0.98	$1.40
Minority	$0.41	$0.58
No: Price to all	$0.70	$1.00

Note: The dollar values in this exhibit are drawn from the calculations in Exhibit 15.3. Note the correspondence of this exhibit with Exhibit 15.2: The southeast corner corresponds to the base case—the value for the base case is simply the total value, $100, divided by the number of shares, 100, given in the assumptions.

WHERE DO ILLIQUIDITY DISCOUNTS COME FROM?
LIQUIDITY IS AN OPTION

First we turn to a consideration of the value of liquidity. This has been the focus of considerable research and recently the useful application of an options perspective.

Liquidity Defined

Illiquidity, or lack of marketability of an asset, commands a discount sufficient to induce investors to buy the nonmarketable asset rather than an identical marketable asset. "Liquidity" and "marketability" are often used interchangeably. However, the terms differ in subtle ways. Liquidity is the ability to exit rapidly, to find a ready price and counterparty. Marketability, on the other hand, is the right to sell (i.e., legally or under the terms of a contract). An asset could be marketable, but not liquid: You may have the legal right to sell a toxic waste dump, but may not find any buyers.

The distinction is crucial for owners of *letter stock*,[4] shares acquired in a private placement of equity under Rule 144 of the SEC. Letter stock is not marketable during the first year after investment. However, the issuer may be publicly listed for trading and generally have a liquid market in its shares. Thus, letter stock issued by this company could be liquid but not marketable. For simplicity of presentation in this chapter, "liquidity" is used in the generic sense of being able to sell. But in specific situations, the M&A professional should determine with competent legal counsel whether assumptions of marketability and liquidity might differ.

Empirical Research on Illiquidity Discounts

Research on government debt, currency options, letter stock, and initial public offerings tells us that liquidity is valuable. For instance, the more liquid Treasury bills offer yields 35 basis points lower than the less liquid Treasury notes.[5] A similar study[6] of Japanese government debt finds a yield difference of 50 basis points. An analysis[7] of liquidity in the euro corporate bond market finds yield differences of as much as 47 basis points. And in the currency options market, the more liquid exchange-traded options sell for about 25 percent more than the less liquid over-the-counter currency options.[8] Equity-linked bonds in the United Kingdom provided the same payoff as investment in an equity index, but were relatively less liquid. Dimson and Hanke (2001) found that over 1989 to 2001 the equity-linked bonds traded at an average 3.35 percent discount to the index.

But of greatest relevance to the analysis of M&A transactions involving illiquid securities is the research on five topics:

1. Discounts associated with *letter stock*, as compared to liquid shares in the market. Studies of letter stock discounts are the most popular points of reference for practitioners. Exhibit 15.5 summarizes these studies and shows average discounts ranging from 13 percent to 45 percent. Silber (1991) finds discounts as high as 84 percent. Finnerty (2002) reports an enormous range, from –47.17 percent (i.e., a *premium*) to 68.3 percent. Plainly, discounts on letter stock vary widely.

EXHIBIT 15.5   Research on Letter Stock Liquidity Discounts

Study	Observations and Time Period	Mean Discount
Studies by Scholars		
Wruck (1989)	N = 99, 1979–1985	13.5%
Silber (1991)	N = 69, 1981–1988	33.75%
Hertzel and Smith (1993)	N = 106, 1980–1987	20.14%
Longstaff (1995)	N/A*	25–35%*
Finnerty (2002)	N = 101, 1991–1997	20.13%
Studies by Government		
SEC (1971)	N = 398, 1966–1969	25.8%
Studies by Practitioners		
Gelman (1972)	N = 89, 1968–1970	33%
Moroney (1973)	N = 146, N/A	35.6%
Trout (1972)	N = N/A, 1968–1972	33.5%†
Maher (1976)	N = N/A, 1969–1973	35.4%
Standard Research Consultants (1983)	N = N/A, 1978–1982	45%†
Willamette Management Associates‡	N = N/A, 1981–1984	31.2%†
Hall and Polacek (1994)	N = 100, 1979–1992	23%
Oliver and Meyers (2000)	N = 53, 1980–1996	27%
Johnson (1999)	N = 72, 1991–1995	20%
Aschwald (2000)	N = 23, 1996–1997	21%
	N = 15, 1997–1998	13%

*Longstaff's result is the estimated *maximum* discount for nonmarketability.
†Median values.
‡Cited in Pratt (1989).

2. Discounts associated with *entrepreneurs' restricted shares.* Founders and managers of companies can be restricted from selling their stock, due to the terms of executive compensation schemes or IPO stock lockups. Illiquidity combined with a lack of portfolio diversification for their personal wealth can impose sizable discounts. Kahl, Liu, and Longstaff (2001) modeled the discounts and found that where stock is restricted for five years and it represents 50 percent of the entrepreneur's wealth, the illiquidity discount could vary between 20 and 70 percent. They find that volatility of stock price and length of restriction period are key drivers of the discount.

3. Discounts implied in *private placements before public transactions.* The letter stock studies consider private placements for securities of public firms. But private placements for private firms grant a different perspective on illiquidity. Emory (2000) found discounts over 1981–2000 averaging 47 percent. Willamette Management Associates, cited in Pratt (2001), found average annual discounts clustering in the 45 to 50 percent range. These pre-IPO transactions are often with insiders; it is possible that special influence or use of the form of these transactions as a form of executive compensation may confound inferences about liquidity.

4. *IPO underpricing and flotation costs* give another perspective on illiquidity. The cost of going public is the price a firm pays to achieve liquidity and other aims.[9] These costs consist of direct costs (i.e., the gross underwriting spread) and indirect costs (the underwriting discount).[10] The literature on these costs is extensive[11] and finds direct costs of about 7 percent and indirect costs of about 15 percent, yielding a total cost of about 22 percent. Like the pre- versus post-IPO comparison, using IPO costs as a measure of liquidity discounts suffers from selection bias: Only the successful issuers are observed; ignored are those firms that must—or choose to—remain private.

5. Comparison of *acquisitions of similar public and private firms* matched for size, industry, and time period. Using a multiples-based approach, Koeplin, Sarin, and Shapiro (2000) estimated an "as-if public" valuation for acquisitions of private firms, 84 in the United States and 108 outside, between 1984 and 1998. Then using the actual transaction prices, they calculated the discount from this public value. Based on EBIT and EBITDA multiples, they found an average discount of 20 to 28 percent for U.S. firms and 44 to 54 percent for foreign firms. Several studies find a sizable announcement day return to bidders when they buy private firms as opposed to public firms.[12] Chang (1998) finds a positive 2.64 percent cumulative average return to bidders who buy private targets with stock. The return in the cases where a new significant shareholder is created in the deal is positive 4.96 percent. Chang hypothesizes that the new block holder will help to monitor the public firm's management. Hansen and Lott (1996) report that in buying a private firm, bidders earn a 2 percent higher cumulative average residual (CAR)[13] than when buying a public firm. Fuller, Netter, and Stegemoller (2002) report a 3.08 percent higher CAR for acquisitions of private companies. Explanations by researchers point to bargaining advantages by public buyers of private firms, the absence of competitive bidding that creates favorable purchase prices, and the creation of new power groups in the buyer company that will motivate the buyer to perform well.

In sum, empirical research finds that illiquidity commands a discount. However, there is little agreement about its size. This is probably due to the variation in kinds of securities, their issuers, government regulations (such as the reduction in the letter stock holding period) and market conditions (such as the opening and closing of the IPO window). Sziklay (2001) summarizes a wide range of factors that practitioners believe to explain cross-sectional variations in letter stock discounts: the size of issue, the time or expense involved in reselling the stock, the existence of a liquid market for the restricted stock, and the size and profitability of issuer.

The Concept of Liquidity as an Option

Options-based thinking provides a framework that can help to guide the practitioner through the range of empirical findings. The right to exit promptly from an investment is equivalent to a put option, of which two drivers are important to the practitioner:

1. *Uncertainty.* The greater the volatility in the value of the underlying stock, the greater will be the value of liquidity. Stated alternatively, the greater the uncertainty, the greater will be the discount for illiquidity.

2. *Time.* The longer the delay in exiting from an investment, the greater will be the discount for illiquidity.

Liquidity discounts have been modeled using option pricing theory. Alli and Thompson (1991) estimated the value of liquidity as the value of a European put option with a strike price equal to the share price at date of issue. Chaffe (1993) applied the put option model to the liquidity discount in private company valuations. Longstaff (1995) estimated the analytical upper bound of the value of liquidity as the price of a lookback option. He reported discounts in the range of 25 to 35 percent given typical liquidity restrictions on private placements. Exhibit 15.6 gives selected maximum liquidity discounts implied by his model. Plainly, volatility and time explain wide variation in discounts.

Finnerty (2002) extended this options-based view with a cross-sectional analysis of letter stock discounts. He found that volatility, the length of the restriction period, the riskless rate, and the stock's dividend yield significantly determine the discount. Dividend payments dampen the size and variability of the discount. Other factors he noted are information and the effect on equity ownership concentration. He uses his model to assess the actual premiums, and finds that the options-based model describes well actual premiums that are within a reasonable middle range of volatility (i.e., between 30 and 70 percent). But actual premiums are *overstated* when volatility is low (i.e., under 30 percent), and *understated* when volatility is high (i.e., over 70 percent). This result is consistent with blind application in practice of a fixed discount regardless of risk.

WHERE DO CONTROL PREMIUMS COME FROM? CONTROL IS AN OPTION

In this section, the spotlight shifts to the valuation of control. Here, too, an options perspective lends useful traction to the analyst.

Control and Control Premium

"Control" is the *right* to direct the strategy and activities of the firm, to allocate resources, and to distribute the economic wealth of the firm. Defined in the sense of

EXHIBIT 15.6 Longstaff's Upper Bounds for Percentage Discounts Because of Lack of Marketability (Percentage Discounts from Marketable Values)

Marketability Restriction Period	Volatility = 10%	Volatility = 20%	Volatility = 30%
180 days	5.768	11.793	18.082
1 year	8.232	16.984	26.276
2 years	11.793	24.643	38.605
5 years	19.128	40.979	65.772

Source: This is a small subset of results from Longstaff (1995), Table II.

rights, control is a call option on alternative strategies and policies of the firm. Thinking of control this way yields two fundamental ideas:

1. *The value of control is contingent, not fixed.* When the current strategy is working well, the option to switch strategies is out of the money. When the current strategy is working poorly, the option to switch will be in the money. Thus, the value of control will vary, depending on the economic success of the current strategy.
2. *The drivers of the value of control are based on the volatility of those values,* for the firm under current and alternative strategies, and the uncertainty or volatility of those values. This suggests that control will be worth more the greater the uncertainty.

Following this logic, "control premium" is the *price* of the control right. In casual conversation, for instance, one often hears "control premium" used to describe the *purchase premium*[14] with which the buyer induces the seller to sell. It is inappropriate to mingle the two ideas or to use the purchase premium as a proxy for the control premium. One should not use the average purchase premium for a sample of companies as the basis for recommending a premium for control.[15] The purchase premium reflects both the value of the control right and the value of expected synergies.

Where one shareholder has controlling power and the others do not, the value of the controller's equity interest will rise by the control premium; the value of the minority shareholders will suffer a minority discount—this is illustrated in the examples given in Exhibits 15.3 and 15.4. The wealth transfers resulting from the changes in the distribution of controlling power among shareholders are a prime reason for studying the value of control.

Control Right Is Derived from *Relative* Power

A simplistic view is that *controlling power* is conferred by owning or being able to direct the votes of 50.1 percent or more of the firm's shares. However, when shares are widely dispersed among shareholders, none of whom own more than 50 percent of the stock, effective control may be achieved with a block of shares of as little as 20 or 30 percent. The issue is not simply the size of the voting groups, but rather how often any of those groups might become *decisive* in the event of a vote. Once you think in terms of winning shareholder votes, you begin to grasp that *voting power is contingent*—votes are relevant only in the context of some game. This can be illustrated by calculating an index for voting power, called the Shapley Value. This value measures the number of times each player in a contest will be pivotal to the voting outcome. Power is found to be a nonlinear function of votes—this is the breakthrough insight of Lloyd Shapley.[16] A related insight is that the percentage of the shares that is truly dispersed (i.e., "atomistic" or "free-floating") is an important determinant of the control contest—quite simply, the distribution of votes prophesies outcomes.

The *Shapley Value* (SV_i) is the ratio of the number of combinations of voting groups in which shareholder i is pivotal to the outcome, divided by the number of all possible combinations. To be "pivotal" is to decide the outcome of the voting contest:

$$SV_i = \frac{n_i^{\text{Pivotal}}}{n!} \qquad (4)$$

The larger the Shapley Value is, the more powerful is shareholder i. Intuition suggests that the more votes shareholder i has, the more likely that shareholder will be pivotal. Voting power is generally related to the number of votes one has. But how the rest of the votes are distributed among voters also affects the power of the individual shareholder. Here's where the measurement of the Shapley Value becomes complicated to model (and beyond the scope of this discussion).[17] Nevertheless, the insights that the Shapley Values afford about voting power are fascinating. Consider, for instance, a setting in which there are two competing raiders soliciting proxies for a takeover of one target. The question is, how powerful is the "ocean" of atomistic voters? The atomistic voters are all the non-aligned shareholders—the use of "atomistic" is game theory jargon to suggest that none of these voters is *individually* powerful. The big insight of the game modeling is that these voters can become very powerful as a group in some circumstances. Exhibit 15.7 presents Shapley Values for the ocean of atomistic voters over a range of scenarios in a setting where two larger shareholders are competing for control, such as a proxy contest. The atomistic shareholders are relatively powerful in the absence of powerful voting blocks—see the northwest corner of the table where each of the control-seeking shareholders or proxy contestants has only 10 percent of the votes; there, the atomistic voters are most powerful. As the proxy contestants gain votes, the power of the atomistic voters subsides. Generally, the more votes you have, the more powerful you are. But there is an interesting exception to this rule: in the southeast corner of the table, the power of the atomistic voters rises sharply. This is consistent with intuition. If you are the swing voter in a contest, even though you may have relatively few votes, you can be powerful.

EXHIBIT 15.7 Sensitivity Analysis of Shapley Values for the Ocean in a Hypothetical Proxy Contest

| Votes of Control	Votes of Control Shareholder #1			
Shareholder #2	10	20	30	49
10	0.78	0.65	0.50	0.05
20	0.65	0.56	0.48	0.06
30	0.50	0.48	0.50	0.09
49	0.05	0.06	0.09	0.50

Comment: In this table, the higher the Shapley Value, the greater the power of the oceanic voters. The model assumes 100 votes are outstanding. Each cell estimates the power of the atomistic voters where the number of votes held by each of the competing proxy contestants is indicated in the row and column headings.
Source: See the spreadsheet file "Power.xls" on the CD-ROM.

Potential Private Benefits May Drive the Value of Control

One reason that control might be valuable is that it presents the opportunity for the majority to expropriate wealth of the minority. Thus, control confers the option to steal. Benefits not shared by all shareholders are private benefits. Dyck and Zingales (2001) examined a large sample of M&A transactions across 39 countries and found that the premium paid for control is higher in countries that protect investors less and thus permit extraction of private benefits. An extension of the private benefits findings is an emerging body of research on forms of intercorporate investing that achieve effective voting control. Examples of these forms are cross-shareholding arrangements and *pyramid* arrangements. The concern is that controlling corporate shareholders might expropriate wealth of the minority (called "tunneling").

Pyramids are a way to extract private benefits. Bebchuk, Kraakman, and Triantis (1998) showed that for relatively small investments in a pyramidal firm or a group with extensive cross-shareholdings, a controller might gain control rights disproportionately greater than cash flow rights from any one of the individual enterprises in the group or pyramid. Exhibit 15.8 illustrates that if it takes only a 25 percent voting block to control a corporation, then an initial investment can be leveraged rapidly to control value that is many times the initial outlay. Marco and Mengoli (2001) found that stock pyramiding among Italian firms is associated with wealth transfers toward entities located at higher levels of the organization. The wealth recipients reported significantly positive CARs; the minority reported losses. Similar results were reported for firms in Korea (Bae, Kang, and Kim 2002), China (Liu and Lu 2002) and India (Bertrand, Mehta, and Mullainathan 2000). Parsons, Maxwell, and O'Brien (1999) found that the rise of major investors in a number of
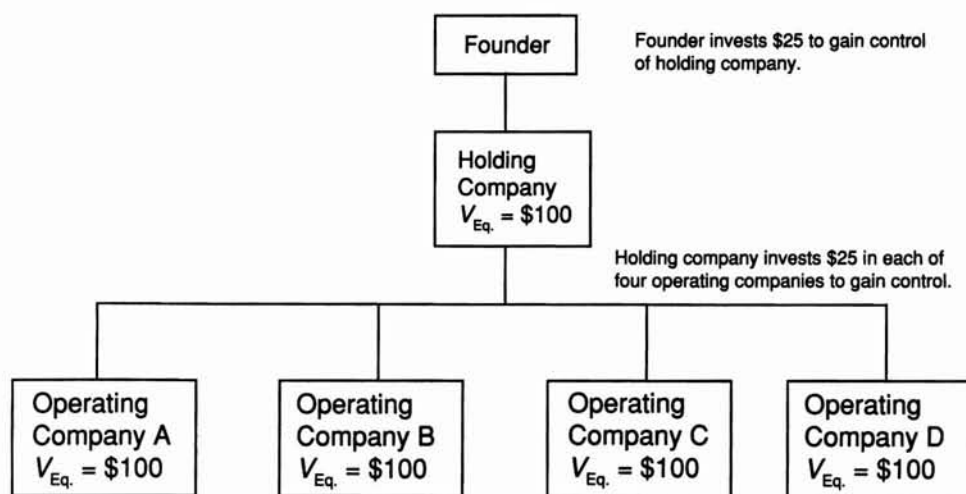
Result: With an investment of $25, founder controls operating companies with a total value of $400. This type of leverage increases as the percent of votes necessary to control a firm decreases.

EXHIBIT 15.8   Simple Illustration of the Control Economics of a Pyramid Holding Company

firms in the same industry can change the competition and move the industry toward monopoly.

Strategic Flexibility May Drive the Value of Control

A complementary hypothesis to private benefits (the option to steal) is that control confers the option to direct the strategy of the firm in ways that always maximizes value for shareholders. The controller of this corporation always does what is economically right. Where the expropriation of private benefits always results in a wealth transfer from the minority to the controller, the strategic flexibility aspect of control increases the value of the whole firm. Control is a right to determine the future strategy of the firm, a switching option. Margrabe (1978) and others have explored the valuation of switching options in industrial settings.[18] While Myers (1977, 1984), Kester (1981, 1984), and others studied the value of rights to decide in corporate resource allocation decisions, rights to control the entire enterprise remain relatively unexplored. An example of a controlling shareholder who uses that control not to steal but rather to exercise wisely the rights of strategic direction would be Warren Buffett, CEO of Berkshire Hathaway. Arguably, Buffett has done what is economically right, since for the past 40 years an investment in Berkshire Hathaway has beaten the appropriate investment benchmarks by a wide margin.

What is interesting about strategic flexibility as a driver of the value of control is what happens to the value of a firm when the right to change strategies is constrained or squandered. The overarching power of unions (as in the case recently of United Airlines), or a fixed commitment to obsolete technology (the U.S. integrated steel industry), or a dedication to "the way we've always done things" (the mom-and-pop retail establishment) would be examples of enterprises without strategic control. Such enterprises should sell at a discount compared to firms that have and use strategic flexibility.

The concept of strategic flexibility expands our understanding of control beyond insights afforded by the private benefits hypothesis. In a world where control always leads to the expropriation of private benefits, it will be true that the controller's gain equals the minority's loss. But strategic flexibility enriches that story; with flexibility, it *may* be possible that the controller does not expropriate private benefits, but rather, runs the firm in the interests of all owners—this is what the legal systems in most developed countries seek to promote.

Empirical Findings on the Value of Control

Research tells us that owning a controlling interest commands a premium; owning a minority (i.e., opposite of controlling) interest commands a discount relative to the controller.

RESEARCH ON DUAL-CLASS SHARES   Studies of firms that have two classes of common stock outstanding show that the class with superior voting rights trades at a material premium relative to the other class. These *dual-class shares* structures arise as antitakeover devices or where a founding family seeks to exert control of a firm with a large shareholder base. In 1999, 219 out of 1,900 large publicly traded firms in the United States had dual-class structures.[19] Bergstrom and Rydkvist (1990)

note that over 70 percent of the firms listed on the Stockholm exchange had dual-class structures in the late 1980s. Zingales (1994) reported that 40 percent of the firms listed on the Milan exchange had dual-class structures. And Hauser and Lauterbach (2000) note that 40 percent of the firms on the Tel Aviv stock exchange had dual-class structures in 1989.

Exhibit 15.9 summarizes the findings of 11 studies of the premium at which senior voting shares traded over junior shares and shows a significant but widely varying premium between 5 and 80 percent. Hauser and Lauterbach (2000) found that reversions by dual-class firms back to one share, one vote structures were accompanied by positive excess returns. Bruner (1999) found that in the case of Renault's attempted acquisition of Volvo, Volvo's voting premium fell from 46.6 percent to 2.3 percent when Renault acquired a significant block of Volvo's stock. Nenova (2003) finds that control premiums in dual-class structures vary significantly across countries according to the legal protections for minority shareholder rights.

Nenova (2001) reports dramatic fluctuations in dual-class premiums in Brazil during a period of market and regulatory reform that strengthened the rights of minority shareholders. A study by Doidge (2003) suggests that variations in shareholder protection among countries may explain some of the variation in the dual-class premium: firms from countries with poor protection to minority investors have higher voting premiums.

RESEARCH ON BLOCK TRADES   The trades of large blocks of stock (where a "block" is commonly defined as in excess of 10,000 shares) can alter the ownership structure of a firm. Barclay and Holderness (1989) studied trades of blocks of more than 5 percent of a public firm's shares and found that the blocks traded at a 20 percent premium relative to the post-transaction price. The authors argued that the premium reflected the voting power of the block.

EXHIBIT 15.9   Summary of Research on the Control Premium of Senior Voting Shares over Junior Voting Shares in Dual-Class Share Structures

Study	Country	Average Premium
Rydkvist (1996)	Sweden	12.0%
Lease, McConnell, and Mikkelson (1983)	United States	5.4%
DeAngelo and DeAngelo (1985)	United States	5.0%
Doidge (2003)	United States (foreign firms cross-listed into the U.S.)	8.0%
Levy (1982)	Israel	45.5%
Biger (1991)	Israel	74.0%
Megginson (1990)	Britain	13.3%
Smith and Amoako-Adu (1995)	Canada	10.4%
Zingales (1994)	Italy	80.0%
Horner (1988)	Switzerland	10.0%
Kunz and Angel (1996)	Switzerland	18.0%

RESEARCH ON M&A TRANSACTIONS   Chapter 3 discusses numerous studies that find sizable premiums paid to shareholders of target firms in M&A transactions. Henry Manne (1965) argued that these premium payments reflect the value of control of the target, that control is valuable, and that an active market for corporate control exists. Hanouna, Sarin, and Shapiro (2000) argued that the true control premium needs to be separated from the premium driven by other considerations (such as synergies). They analyzed a very large sample of transactions comparing those in which the buyer acquired a minority position, versus the ones where the buyer acquired a controlling position. They found that a majority position commands a 20 to 30 percent premium compared to the price paid for a minority position.

If the control right is an option, then the wide variation in control premiums and voting premiums is attributable to the two key drivers of option value: volatility and duration of the right. It remains for empirical researchers to explore the ability of volatility to explain the cross-sectional variation in the value of control.

INTERACTION OF LIQUIDITY AND CONTROL

Under the conventional method, the analyst selects the relevant discount and premium as if they are independent. But there are four reasons why liquidity and control may interact:

1. *Liquidity may bring with it transparency, which may reduce the value of control.* For instance, registration requirements under U.S. law and securities regulations, and listing requirements on the NYSE mandate procedures of governance and reporting that may constrain the ability of controllers to extract private benefits and not operate the firm in the interests of all shareholders. An emerging body of research on the relation between governance and share value underscores the benefits of transparency; see, for instance, La Porta et al. (1999). Lerner and Schoar (2001) argue that the need for control will vary with liquidity.[20] It may be that if shares are highly liquid, investors may have less incentive to oversee firms.[21] A liquidity discount may be more severe in the instance of more asymmetric information (less transparency), such as young firms with no track record and incomplete reporting infrastructure or divided equity investors who don't communicate (e.g. cross-border investors). Lerner and Schoar examine the case of American Research and Development to understand why a venture capitalist would place restrictions on the transferability of partnership interests. Their conclusion is that these restrictions bar "hot money" investors from entering the pool and instead admit only the more patient, well-capitalized investors.
2. *Liquidity may be associated with more dispersed shareholdings.* This may increase the power of controllers (see Milnor and Shapley 1978) and therefore increase their ability to extract private benefits and operate the firm in nonvalue-maximizing ways. This would suggest a direct relationship between liquidity and control.
3. *Control positions tend to be sticky.* Controlling shareholders amass their positions with difficulty, tend not to trade shares actively, and if they decide to sell

control intact, may experience delays in selling. Heaney and Holmen (2002) find that senior voting shares in Sweden are much less liquid than junior shares.

4. *With control, any decision to liquidate is complicated by the right to choose the most attractive strategy.* Thus, liquidation occurs only after you have tried to create value through strategic choice. This time sequencing of control and liquidation choices suggests that the decision to liquidate depends on the prior choice of strategy.

If rights to market and control interact, then they form a *compound* option. This would imply two key insights. First, it is inappropriate to study liquidity in the absence of control, and vice versa. Second, compound options are very difficult to model analytically. As a practical matter, researchers must resort to numerical option pricing methods to explore the valuation of these rights.

Using option pricing methods to value control rights is in its infancy. While the valuation of liquidity rights is a little more advanced, much more work remains to be done before the practitioner will be able to estimate premiums and discounts through option pricing. Still, the early results are promising and consistent with intuition. Bruner and Palacios (2003) estimated the joint discounts from loss of liquidity and control using Monte Carlo simulation.[22] Under conventional assumptions, they obtained estimated discounts in a range consistent with those typically seen in practice: 10 to 50 percent. But importantly, they found that the size of the discounts was very sensitive to time (i.e., length of illiquidity and of minority status) and to uncertainty about the value of the underlying asset (i.e., volatility). Generally, the discounts were large in scenarios of longer time and greater uncertainty. Their modeling offers five insights:

1. *One size does not fit all.* Their modeling confirmed that the use of a standard "haircut" for illiquidity or lack of control might leave money on the table.
2. *Control and liquidity options interact.* These two effects on value are not merely additive.
3. *Volatility is the major driver of discounts.* While volatility cannot be observed, one can "trade on volatility" in negotiations. The options investor can examine options prices for their consistency with the trader's own assessment of the appropriateness of the volatility implied in the price: This is called "trading on volatility."[23]
4. *The options view provides a benchmark* for testing the reasonableness of discounts and premiums.
5. *Lack of control trumps lack of liquidity.* The modeling suggests that if you had to sacrifice one of the two options, you should give up liquidity first. This is because with control you have more flexibility to create value than without.

MINI-CASE: ATTEMPTED ACQUISITION OF VOLVO BY RENAULT, 1993

The attempted merger of Volvo and Renault in 1993 illustrates the possible significance of synergies and discounts for illiquidity and minority status. Building on equation (1), we can think of the postannouncement value per share as a composite

of three quantities: a base case valuation of the firm on an as-is basis, the present value of cash flows arising from synergies, and a premium or discount for change in liquidity and control.

$$P_{After} = \frac{(V_{Base\ case,\ firm\ before} + V_{Synergies})(1 + \%\Delta_{Liquidity\ and\ control})}{Number\ of\ shares_{Before} + Change\ in\ number\ of\ shares} \qquad (5)$$

Modeling the share price in this way emphasizes that the market reaction following the announcements of acquisitions, financings, and restructurings is a reflection of three anticipated changes in the firm: (1) in operations and synergies, (2) in share ownership, and (3) in liquidity and control. This has important implications for the inferences one might draw about the operating benefits derived from these transactions.

The economic motivations for the Volvo/Renault deal were sizable synergies. Bruner (1999) estimated these to have a maximum present value of SEK 17.95 billion, quite large in comparison to AB Volvo's equity value on September 6, 1993, of SEK 37.5 billion. Many investors questioned the estimated synergies, simply on the basis of past experience: Volvo's CEO had consummated previous acquisitions that failed to live up to past expectations. And yet industry experts agreed with the basic logic of consolidation in the auto business: Achieving economies of scale in purchasing and new product development was the sure route to survival and profitability. One reading of the demise of the proposed merger was that investors did not believe in the merger synergies or the expressed motivations for the deal.

Consistent with the framework offered here, a second consideration can complement the analysis of the deal: rights of liquidity and control. The proposed deal would reduce *both* Volvo's control and liquidity of its automotive business. Regarding liquidity, the new firm, Renault-Volvo (RVA), would be privately held by two shareholders (the government of France and the holding company, AB Volvo). The French government had announced that it intended to privatize Renault in 1994, though many observers regarded the timing as figurative—strong unions within Renault, the Socialist party in France, and a French political consensus that favored having a French national champion in the automobile industry would likely delay meaningful privatization.

Regarding control, the government of France would own 65 percent of the new firm, and Renault's executives would dominate the upper ranks of the firm. The French government and AB Volvo agreed not to sell or pledge their respective share holdings until the privatization of Renault-Volvo. Each also agreed to give the other a right of first refusal on the sale of shares, and not to sell shares to a competitor. The French government announced that it intended to privatize Renault-Volvo by selling its shares principally to a circle of friendly French corporations, such as defense contractors and French state-owned banks and insurance companies. The French government would retain an unusual right, a "golden share" that retained for the government the ability to prevent an investor from acquiring (or voting) more than a 17.85 percent direct interest in the new firm. Like a poison pill or control share antitakeover amendment, the golden share could change the voting power of sizable shareholders such as AB Volvo. The French had discretion in using the golden share, however, as the limitation was not automatic. Golden shares have

been a common feature in the privatization of state-owned enterprises. This right would last indefinitely.

In summary, the merger proposal offered Volvo's shareholders participation in the benefits of potential new synergies in exchange for worsened liquidity and control (in technical terms, a short position in a bundle of control options, including the golden share, a privatization option concerning the timing and magnitude of any public offering of RVA shares, as well as an option concerning the targeted purchasers of any shares offered).

Volvo's board had endorsed the merger proposal. From the standpoint of Volvo shareholders, this agreement would sharply limit the liquidity of their investment and their control. Volvo, the holding company, would lose some control from being a full, 100 percent owner of its automotive business to a minority holder of a larger automotive business. The government of France would dominate the shareholder group and determine the date and pricing of its privatization. Even after going public, the government would continue to hold a golden share, in effect, a veto over future strategic decisions of the automotive firm.

Volvo's share prices fell dramatically following the announcement of the merger proposal. Bruner (1999) reports that abnormal returns on September 6–7, 1993, were –6.04 percent for Volvo's superior-voting A shares, and –6.64 percent for the junior-voting B shares. Over the following seven weeks, abnormal returns accumulated to –21.99 percent for the A shares and –22.04 percent for the B shares. This cumulative abnormal return represents a decline in equity value of about SEK 8.3 billion (US$ 1.055 billion). A large portion of this wealth destruction occurred on dates of the release of detailed information about the merger terms.

Securities analysts estimated the market value of AB Volvo's investments (enterprise value) in the automotive business on a stand-alone basis to be SEK 32.92 billion. The projected value of Renault-Volvo without synergies was SEK 85 billion. The 22 percent cumulative market-adjusted loss (SEK 8.3 billion) in AB Volvo shares following the announcement of the deal equates to a discount of 23 percent from the estimated value of Volvo's interest in Renault-Volvo with full synergies (SEK 36.03 billion); assuming zero synergy value, the discount increases to 27.9 percent (on a base value of SEK 29.75 billion). These discounts suggest that the rejection of the deal by Volvo's investors was founded on the expectation of material delays in AB Volvo's ability to liquidate its interest and loss of flexibility to switch strategy.

Were these losses in value consistent with the erosion of Volvo shareholders' liquidity and control rights? Bruner and Palacios (2003) simulated the mean discount for minority control and illiquidity under the assumption of conventional parameters prevailing in 1993 and found that discounts in the range of 22 to 28 percent are fully explained by the loss in control and an illiquidity delay of one year. When illiquidity increases to five years, the discount increases to 39 percent.

The case of Volvo and Renault underscores the potentially large impact of liquidity and control on shareholders' wealth. Also, option valuation techniques afford a benchmark test of reasonableness for observed discounts. Finally, as noted earlier the analyst should take care in assessing the purchase premium: It is an amalgam of synergy value and discounts or premiums for liquidity and control effects.

CONCLUSION

This chapter has explored some of the properties of liquidity and control rights in M&A and illustrated the traditional multiplicative method for adjusting for premiums and discounts.

- Illiquidity and control asymmetry can affect transaction prices. These effects are traditionally modeled multiplicatively. The example offered here showed that illiquidity and control produced sharply different share values from the base case.
- The traditional approach draws on prices in comparable transactions. Option pricing techniques (especially simulation) can offer a benchmark test of reasonableness of premiums and discounts, though the application of these techniques in this area is still in its infancy.
- Liquidity and control are options, driven significantly by uncertainty about the value of the underlying assets.
- The value of these rights varies. The optionality in these rights helps explain the range of findings about their effect on value of equities. Volatility and time produce material variations in estimates of the premiums and discounts. It remains for future research to enhance the ability of practitioners to estimate the discounts for illiquidity and control. One property of liquidity and control rights has received little attention: the interaction between these rights. The interaction arises because these rights can combine to form a compound option.

NOTES

1. The alternative would be an *additive* model in which the effects of liquidity and control would simply be summed. The multiplicative approach is more consistent with the interaction between liquidity and control rights—this interaction is discussed in the section on Volvo and Renault later in this chapter.
2. Instead of adjusting the purchase price, one could adjust the discount rate in a DCF analysis to account for the value effects of liquidity and control. For instance, Arzac (1996) suggests that the premium to be added to the discount rate can be estimated using the following formula, where d = illiquidity discount, k = cost of equity or WACC, and g = perpetual growth rate:

$$\text{Risk premium for illiquidity} = \frac{d(k-g)}{(1-d)}$$

   Whether one adjusts the discount rate or the total value of the firm for illiquidity, one must still have an estimate for d, the illiquidity discount. Pratt's approach gives a more transparent presentation of the effect of these adjustments and therefore seems more useful in the context of negotiation, deal design, and communication with investors.
3. The alert reader will notice the semantic emphasis here. This chapter defines "premium" and "discount" relative to a base case valuation. Some analysts

might casually define a control premium relative to the value of minority shares, rather like a percentage spread between shares with voting rights and those without. The problem with this second definition is that in most settings the value of control and minority shares is what you are trying to determine. Thus, it makes no sense to define a control premium over the junior shares. The semantic emphasis here is for analytic convenience, since one generally can get a base value of the firm as defined here. As the text emphasizes, all discounts and premiums start from a base.

4. Letter stock derives its name from the requirement that investors in securities issued under Rule 144 of the SEC must certify that their investment will be held and not resold. Before 1997, the SEC imposed a two-year minimum holding period on letter stock after which the stock could be sold with certain restrictions; in the third year, the stock could be sold without restrictions. On February 20, 1997, the SEC reduced the minimum holding period to one year, and the unrestricted holding period to two years.

5. See studies by Amihud and Mendelson (1991) and Kamara (1994).

6. See Boudoukh and Whitelaw (1991).

7. See Houweling, Mentink, and Vorst (2002).

8. See Brenner, Eldor, and Hauser (2001).

9. Indirect IPO costs also reflect investors' expectations about the profitability of future investment by the issuer. Therefore using IPO costs possibly confounds marketability with other effects.

10. The gross spread is the sum of the management fee, underwriting fee, and selling concession as a percentage of the amount offered. Indirect costs are measured by initial returns or underpricing (i.e., the day n close price divided by the offer price minus 1).

11. The following sources offer a cross section of the research on IPO costs: Chaplinsky and Ramchand (2000); Chen and Ritter (2000); Loughran and Ritter (2002); Loughran, Ritter, and Rydkvist (1994); Ritter (1987); and Ritter (1984).

12. Chang (1998) finds a positive 2.64 percent cumulative average return to bidders who buy private targets with stock. The return in the cases where a new significant shareholder is created in the deal is positive 4.96 percent. Chang hypothesizes that the new block holder will help to monitor the public firm's management.

13. Cumulative average residuals are the accumulated daily excess returns over a benchmark like the return on the equity market portfolio. These useful statistics are measures of wealth creation or destruction around an announcement; hence, they are often called "event returns." See Chapter 3 for further discussion of CARs.

14. Purchase premium is typically estimated as the bid price divided by the prebid price minus 1.

15. This practice is, unfortunately, common. For more discussion on the inappropriateness of this practice, see Pratt (2001), pages 33 to 36 and 317.

16. A complete discussion of the analytics of power in the framework of Shapley is beyond the scope of this chapter. See Milnor and Shapley (1978) for a full exposition. The mathematical literature on power is extensive.

17. A fuller expression of equation (3) is this:

$$SV_i = \left(\frac{1}{n!}\right)\sum_{t=1,\ldots,n} (t-1)!\,(n-t)!\,k_t$$

where n is the total number of possible combinations, t is the number of share-holders in winning coalitions that feature shareholder i as pivot, and k_t is the number of times shareholder i is pivotal in winning coalitions of t shareholders.

18. See, for instance, Kogut and Kulatilaka (1994), Kulatilaka (1993), Triantis and Hodder (1990), Trigeorgis (1996), Trigeorgis and Mason (1987), and Upton (1994).

19. V. Rosenbaum, *Corporate Takeover Defenses*, Washington D.C.: Investor Responsibility Research Center Inc., 1999.

20. In a related study, Lerner and Schoar (2002) find that the reverse is also true: The need for liquidity will vary with control and transparency that private equity investors have with regard to their portfolio firms—liquidity becomes a variable of choice, a method of screening out investors who don't have deep pockets.

21. There may be less incentive or greater barriers to monitoring. Grossmann and Hart (1980) argue that it may be harder to exercise effective capital market discipline in cases of highly liquid shares held by widely dispersed (atomistic) shareholders.

22. This follows the work of Boyle (1977) and others who have used simulation to value complex options.

23. To "trade on volatility" is to make investment (buy or sell) decisions based on the volatility implied in options prices. One compares implied volatility to the volatility that would be justified by one's view of the asset.